ABSTRACTS FOR THE CONFERENCE ON "RECENT DEVELOPMENTS IN NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS"

1. **John Lewis, University of Kentucky, USA**

Abstract. **On the Brunn Minkowski Inequality and a Minkowski Problem for Nonlinear Capacitiies**

In this talk we discuss two classical problems in convex geometry. The first problem is a Brunn-Minkowski inequality for a nonlinear capacity, $\text{Cap}_{\mathcal{A}}$:

$$[\text{Cap}_{\mathcal{A}}(\lambda E_1 + (1-\lambda)E_2)]^{\frac{1}{(n-p)}} \geq \lambda\, [\text{Cap}_{\mathcal{A}}(E_1)]^{\frac{1}{(n-p)}} + (1-\lambda)\, [\text{Cap}_{\mathcal{A}}(E_2)]^{\frac{1}{(n-p)}}$$

when $1 < p < n, 0 < \lambda < 1$, and E_1, E_2 are convex compact sets with positive $\mathcal{A}$ capacity. In the second part of this talk we discuss a Minkowski existence problem for a certain measure associated with a compact convex set E with nonempty interior and its $\mathcal{A}$ harmonic capacitary function in the complement of E. If μ_E denotes this measure, then the Minkowski problem we consider in this setting, is that for a given finite Borel measure $\tilde{\mu}$ with support in the unit sphere of Euclidean n space, find necessary and sufficient conditions for which there exists E as above with $\mu_E = \tilde{\mu}$.

2. Lucio Boccardo, University of Roma, Italy

Abstract. **Dirichlet problems with singular convection (or drift) terms**

In a paper ([1]), dedicated to the memory of Guido Stampacchia in the thirtieth anniversary of his death, I improved some of his results (see [8]) concerning the linear Dirichlet problem

$$\begin{cases} -Lu = E(x)\cdot\nabla u + f(x) \ in\Omega \\ u = 0 \ on \ \partial\Omega \end{cases} \tag{2.1}$$

Here Ω is a bounded, open subset of $\mathbb{R}^N$, $N > 2$, $E \in (L^N(\Omega))^N$, $f \in L^m(\Omega)$, $1 \le m < \frac{N}{2}$ and $M(x)$ is a bounded and measurable elliptic matrix. To be more precise, in [1] is proved the existence of u

$$\bullet \begin{cases} \text{weak solution belonging to } W_0^{1,2}(\Omega)\cap L^{m^{**}}(\Omega), \text{ if } m \ge \frac{2N}{N+2}; \\ \text{distributional solution belonging to } W_0^{1,m^*}(\Omega), \text{ if } 1 < m < \frac{2N}{N+2}; \end{cases} \tag{2.2}$$

the result u bounded needs a slightly stronger assumption. Note that the above existence results are exactly the results proved with $E = 0$ in [8] and [6].

The main difficulty is due to the **noncoercivity in** $W_0^{1,2}(\Omega)$ **of the differential operator** $-div(M(x)\nabla v) + div(v\,E(x))$.

• Then, for E belonging only to L^2, a more general definition of solution is useful in order to show the existence.

• Existence results if the principal part in nonlinear can be found in [2] and [7].

2.1. **Work in progress (singular drift terms).** We are working on the boundary value problem

$$\begin{cases} -Lu = E(x)\cdot\nabla u + f(x) \ in\Omega \\ u = 0 \ on \ \partial\Omega \end{cases} \tag{2.3}$$

Of course, the above Dirichlet problem with drift term is classical (since the papers by Stampacchia and Bottaro-Marina), but only • in [4] a complete theory for linear elliptic equations with discontinuous coefficients and singular drift is studied.

Even if this problem is the dual of (2.1), our **nonlinear approach** and our nonlinear estimates do not allow a direct study with the of the duality theory.

• Existence results if the principal part in nonlinear can be found in [7] and [5].

References

[1] L. Boccardo: Some developments on Dirichlet problems with discontinuous coefficients; Boll. Unione Mat. Ital. 2 (2009), 285–297.
[2] L. Boccardo: Finite energy solutions of nonlinear Dirichlet problems with discontinuous coefficients; Boll. Unione Mat. Ital. 5 (2012), 357–368.
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[5] L. Boccardo: work in progress.
[6] L. Boccardo, T. Gallouët: Nonlinear elliptic equations with right-hand side measures, Comm. Partial Differential Equations 17 (1992), 641–655.
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3. Cherif Amrouche, University of Pau, France

Abstract. **Elliptic Problems in Smooth and Non Smooth Domains**
We are interested here in questions related to the regularity of solutions of elliptic problems

$$\operatorname{div}(A\nabla\, u) = f \quad \text{in} \quad \Omega$$

with Dirichlet or Neumann boundary condition. For the last 20 years, lots of work has been concerned with questions when A is a matrix or a function and Ω is a Lipschitz domain. We give here some complements and we extend this study to obtain regularity results for domains having an adequate regularity.

Using the duality method, we will then revisit the work of Lions-Magenes, concerning the so-called very weak solutions, when the data are less regular. Thanks to the interpolation theory, it permits us to extend the classes of solutions and then to obtain new results of regularity.

4. Juha Kinnunen, Aalto University, Finland

Abstract. **Supercaloric functions for the porous medium equation.**
We consider a class of nonnegative supersolutions to the porous medium equation

$$u_t - \Delta(u^m) = 0,$$

in the slow diffusion case $m > 1$. These supersolutions are defined as lower semicontinuous functions obeying comparison principle and they are called m-supercaloric functions. For the ordinary heat equation we have supercaloric functions.

The leading example of a m-supercaloric function with a point singularity is the Barenblatt solution, which corresponds to the fundamental solution for the heat equation. In the slow diffusion case so-called friendly giant and other examples constructed by separation of variables are included in the theory. Their infinity sets occupy a whole time slice and they may blow up arbitrarily fast near the polar set.

This talk reviews the definition and properties of m-supercaloric functions. Connections to weak supersolutions and measure data problems are also explained. We show there are two mutually exclusive alternatives: every nonnegative m-supercaloric function has either a Barenblatt type behavior or blows up at least with the rate given by the friendly giant. Several characterizations for both cases are given.

5. Bernd Kawohl, University of Cologne, Germany

Abstract. **On the geometry of the p-Laplace Operator**
The p–Laplacian operator $\Delta_p u = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is not uniformly elliptic for any $p \in (1,2) \cup (2,\infty)$ and degenerates even more when $p \to \infty$ or $p \to 1$. In those two cases the Dirichlet and eigenvalue problems associated with the p-Laplacian lead to intriguing geometric questions, because their limits for $p \to \infty$ or $p \to 1$ can be characterized by the geometry of Ω. In my little survey I recall some well-known results on eigenfunctions of the classical 2-Laplacian and elaborate on their extensions to general $p \in [1,\infty]$. I report also on results concerning the normalized or game-theoretic p–Laplacian

$$\Delta_p^N u := \tfrac{1}{p}|\nabla u|^{2-p}\Delta_p u = \tfrac{1}{p}\Delta_1^N u + \tfrac{p-1}{p}\Delta_\infty^N u$$

and its parabolic counterpart $u_t - \Delta_p^N u = 0$. These equations are homogeneous of degree 1 and Δ_p^N is uniformly elliptic for any $p \in (1,\infty)$. In this respect it is more benign than the p-Laplacian, but it is not of divergence type.

6. Miroslav Bulicek, Charles University, Czech Republic

Abstract. **Nonlinear ellitpic equations beyond the natural duality pairing**
Many real-world problems are described by nonlinear partial differential equations. A promiment example of such equations is nonlinear (quasilinear) elliptic system with given right hand side in divergence form *div f* data. In case data are good enough (i.e., belong to L^2), one can solve such a problem by using the monotone operator therory, however in case data are worse no existence theory was available except the case when the operator is linear, e.g. the Laplace operator. For this particular case one can however establish the existence of a solution whose gradient belongs to L^q whenever f belongs to L^q as well. From this point of view it would be nice to have such a theory also for general operators. However, it cannot be the case as indicated by many counterexamples. Nevertheless, we show that such a theory can be built for operators having asymptotically the radial structrure, which is a natural class of operators in the theory of PDE. As a by product we develop new theoretical tools as e.g., weighted estimates for the linear problems and the new compensated compactness method represented by the div-curl-biting-weighted lemma.

7. Jose Miguel Urbano, University of Coimbra, Portugal

Abstract. **A proof of the $C^{p'}$-regularity conjecture in the plane**
We establish a new oscillation estimate for solutions of nonlinear partial differential equations of degenerate elliptic type, which yields a precise control on the growth rate of solutions near their set of critical points. We then apply this new tool to prove the planar counterpart of the longstanding conjecture that solutions of the degenerate p-Poisson equation with a bounded source are locally of class $C^{1,1/p-1}$, an optimal regularity result. This is a joint work with Eduardo Teixeira (University of Central Florida, USA) and Damião Araújo (UFPB, Brazil).

8. S. Prashanth, TIFR CAM, India

Abstract. **Criticality theory for Schrodinger operators with singular potential**
In this talk we explain a criticality theory based classification of a large class of Schrödinger operators $-\Delta + V$ with singular potential V. The Harnack inequality may not apply in general for such class of operators.

9. Sun-Sig Byun, Seoul National University, Korea

Abstract. **Optimal regularity results for nonlinear parabolic problems with measure data**
We consider nonlinear parabolic equations having a measure in the right hand side. and prove some higher integrability results of the gradient of solutions to such measure data problems with measurable nonlinearities on nonsmooth bounded domains in the setting weighted Orlicz spaces.

10. **Verena Bogelein, University of Salzburg, Austria**

ABSTRACT. **A variational approach to the porous medium equation**
n this talk we establish an existence theory for the porous medium equation

$$\partial_t u^m - \Delta u = 0, \tag{10.1}$$

and more generally, for doubly nonlinear evolution equations of the type

$$\partial_t b(u) - \text{Div } Df(Du)) = 0, \tag{10.2}$$

where f is only assumed to be coercive and convex. Doubly nonlinear equations possess a wide spectrum of applications, for instance in fluid dynamics, soil science and filtration. Our approach is purely variational. We introduce a nonlinear version of the minimizing movement approach that could also be useful for the numerics of doubly nonlinear equations. It is flexible enough to deal also with obstacle problems with low regularity of the obstacle or time dependent boundary data. This is joint work with F. Duzaar (Erlangen), P. Marcellini (Florence), and C. Scheven (Duisburg-Essen).

11. **Ugo Gianazza, University of Pavia, Italy**

ABSTRACT. **A self-improving property of degenerate parabolic equations of porous medium-type**
We show that the gradient of solutions to degenerate parabolic equations of porous medium-type satisfies a reverse Hölder inequality in suitable intrinsic cylinders. We modify the by-now classical Gehring lemma by introducing an intrinsic Calderón-Zygmund covering argument, and we are able to prove local higher integrability of the gradient of a proper power of the solution u. This is a joint work with Sebastian Schwarzacher of Charles University.

12. **Erika Maringova, Charles University, Czech Republic**

ABSTRACT. **Globally Lipschitz minimizers for variational problems with linear growth**
The classical example of a variational problem with linear growth is the minimal surface problem. It is well known that for smooth data such problem posses a regular (up to the boundary) solution if the domain is convex (or has positive mean curvature). On the other hand, for non-convex domains we know that there always exist data for which the solution does exist only in the space BV (the desired trace is not attained). In the work we sharply identify the class of functionals (such that the minimal surface problem is equivalently described by a particular functional from this class) for which we always have regular (up to the boundary) solution in any dimension for arbitrary $C^{1,1}$ domain. Furthermore, we show that the class is sharp, i.e., whenever the functional does not belong to the class then we can find data for which the solution does not exist.

13. **TV Anoop, IIT Madras, India**

ABSTRACT. **A monotonicity property of the first eigenvalue of p-Laplacian.**
We discuss the monotonicity of the first eigenvalue of p-Laplacian with respect to domain variations. More precisely, let B_1 be a ball in $\mathbb{R}^N$ centered at the origin and let B_0 be a smaller ball contained in B_1. For $p \in (1, \infty)$, we show that the first eigenvalue of the p-Laplacian in annulus $B_1 \setminus \overline{B_0}$ strictly decreases as the inner ball moves towards the boundary of the outer ball.

14. **Karthik Adimurthi, Seoul National University, Korea**

ABSTRACT. **Sharp existence results for quasilinear equations with gradient nonlinearity on the right**

We study the existence problem for a class of nonlinear elliptic equations whose prototype is of the form $-\Delta_p u = |\nabla u|^p + \sigma$ in a bounded domain $\Omega \subset \mathbb{R}^n$. Here Δ_p, $p > 1$, is the standard p-Laplacian operator defined by $\Delta_p u = \text{div}\,(|\nabla u|^{p-2}\nabla u)$, and the datum σ is a signed distribution in Ω. The class of solutions that we are interested in consists of functions $u \in W_0^{1,p}(\Omega)$ such that $|\nabla u| \in M(W^{1,p}(\Omega) \to L^p(\Omega))$, a space pointwise Sobolev multipliers consisting of functions $f \in L^p(\Omega)$ such that

$$\int_\Omega |f|^p|\varphi|^p dx \leq C \int_\Omega (|\nabla\varphi|^p + |\varphi|^p) dx \quad \forall\varphi \in C^\infty(\Omega), \tag{14.1}$$

for some $C > 0$. This is a natural class of solutions at least when the distribution σ is nonnegative and compactly supported in Ω. We show essentially that, with only a gap in the smallness constants, the above equation has a solution in this class if and only if one can write $\sigma = \text{div}\, F$ for a vector field F such that $|F|^{\frac{1}{p-1}} \in M(W^{1,p}(\Omega) \to L^p(\Omega))$

15. **Imran Biswas, TIFR CAM, India**

ABSTRACT. **On the rate of convergence for monotone numerical schemes for nonlocal Isaacs equations**

We study monotone numerical schemes for nonlocal Isaacs equations, the dynamic programming equations of stochastic differential games with jump-diffusion state processes. These equations are fully-nonlinear non-convex equations of order less than 2. In our case they are also allowed to be degenerate and have non-smooth solutions. The main contribution is a series of new a priori error estimates: The first results for nonlocal Isaacs equations, the first general results for degenerate non-convex equations of order greater than 1, and the first results in the viscosity solution setting giving the precise dependence on the fractional order of the equation. We also observe a new phenomena, that the rates differ when the nonlocal diffusion coefficient depend on x and t, only on x, or on neither.

16. **Agnid Banerjee, TIFR CAM, India**

ABSTRACT. **Space time strong unique continuation property for nonlocal parabolic equations**

We study the strong unique continuation property backwards in time for the nonlocal equation in $\mathbb{R}^n \times \mathbb{R}$

$$(\partial_t - \Delta)^s u = V(x,t)u, \qquad s \in (0,1). \tag{16.1}$$

Our main result can be thought of as the nonlocal counterpart of the result obtained by C. Poon for the case when $s = 1$. In order to prove our main result, we develop the regularity theory of the extension problem for the equation (16.1). With such theory in hands we establish:

(i) a basic monotonicity result for an adjusted frequency function which plays a central role in this paper
(ii) an extensive blowup analysis of the so-called *Almgren rescalings* associated with the extension problem.

This is a joint work with Nicola Garofalo.